(Time: 3 hours) **Total Marks: 80**

N.B.: (1) Question **No. 1 compulsory.**

(2) Attempt any Three questions from remaining five questions.

Q1 a) Prove that $\left(1+i\sqrt{3}\right)^8+\left(1-i\sqrt{3}\right)^8=-256$ [5]

b) Express the matrix $A=\begin{bmatrix}3 & -2 & 6\\ 2 & 7 & -1\\ 5 & 4 & 0\end{bmatrix}$ as the sum of the symmetric and a skew symmetric matrix. [5]

c) If $p=y^2+z^2,\ q=z^2+x^2, r=x^2+y^2$ *then evaluate* $\frac{\partial(p,q,r)}{\partial(x,y,z)}$. [5]

d) Using Newton-Raphson method for the equation $x^3-2x-5=0$, find the root starting with $x_0=2$ as initial value with an accuracy 0f .0001. [5]

Q2 a) Test for consistency and if possible solve

$x+2y-z=2,\ 3x+8y+2z=10,\ 4x+9y-z=12$ [6]

b) Find all the values of $\left(1-i\sqrt{3}\right)^{\frac{1}{4}}$.

[6]

c) If $u=sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, P.T $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{1}{4}(tan^3u-tanu)$ [8]

Q3 a) Separate into real and imaginary parts $cos^{-1}\left(\frac{3i}{4}\right)$ [6]

b) Find the Rank of the following matrix by reducing to Normal Form

$$A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\end{bmatrix}$$ [6]

c) Examine the function

$f(x,y)=x^3+3xy^2-3x^2-3y^2+4$ for extreme values. [8]

Q4 a) Examine whether the vectors

$X_1 = [1, 1, 1],\ X_2 = [2, 3, 8], X_3 = [1, 2, 3]$ are linearly independent [6]

b) If $sin(\propto + i\beta) = x + iy$, then prove that [6]

$$\frac{x^2}{cosh^2\beta} + \frac{y^2}{sinh^2\beta} = 1 \quad and \quad \frac{x^2}{sin^2\alpha} - \frac{y^2}{cos^2\alpha} = 1$$

c) If $y = cosxcos2xcos3x$ then find n^{th} derivative of y [8]

Q5 a) Apply Jacobi's Iterative method to solve the following equations

$20x + y - 2z = 17,\ \ 3x + 20y - z = -18,\ \ 2x - 3y + 20z = 25$ [8]

b) If $v = (1 - 2xy + y^2)^{\frac{-1}{2}}$ then show that $xv_x - yv_y = y^2v^3$ [6]

c) Expand $log(1 + x + x^2 + x^3)$ up to x^5 [6]

Q6 a) Considering only the principle values, prove that the real part of

$$\left(1 + i\sqrt{3}\right)^{\left(1+i\sqrt{3}\right)} is\ 2e^{\frac{-\pi}{\sqrt{3}}} \cos\left(\frac{\pi}{3} + \sqrt{3}log2\right)$$ [6]

b) If $u = f(x^2 - y^2,\ y^2 - z^2,\ z^2 - x^2)$, then prove that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0.$ [6]

c)Prove that $tan5\theta = \frac{5tan\theta - 10tan^3\theta + tan^5\theta}{1 - 10tan^2\theta + 5tan^4\theta}$ [8]

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